

COMPETITION No.1: THE SOLUTION

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QUESTION: “Every finite set of frequencies belongs to some harmonic series, in that each of the frequencies will be a harmonic of some fundamental frequency.” Is this assertion true? (Explain.) Is there a finite set of frequencies for which such a fundamental does *not* exist? (Explain.) Discuss the musical implications of your findings.

SOLUTION: Yes, *this assertion is true.*

Given any finite set of frequencies $\Omega = \{f_1\text{Hz}, f_2\text{Hz}, \dots, f_n\text{Hz}\}$, define a frequency $f_0\text{Hz}$ such that $f_0\text{Hz} = \text{h.c.f.}\{f_1, f_2, \dots, f_n\}\text{Hz}$. Then the harmonic series of $f_0\text{Hz}$ will incorporate every frequency of Ω . Indeed, $f_0\text{Hz}$ is the *highest* such fundamental frequency. [NB: “h.c.f.” is just mathematical shorthand for ‘highest common factor’.] However, *there are infinitely many finite sets of frequencies Ω for which such a fundamental $f_0\text{Hz}$ does not exist!* I shall furnish one of them below. But to do so, it is necessary first of all to briefly invoke the most current theories of cosmology – cosmology being that field of science which is concerned with the origin, evolution, and fate of our Universe.

Although some eminent cosmologists have recently questioned the efficacy of the Big Bang theory (that the Universe began with some unimaginable cataclysm), it is still widely accepted that physical existence commenced with a “Big Bang” around 15 billion years ago. It is, moreover, generally agreed that the Universe will end sometime in the extremely distant future – although *how* this will transpire is another issue entirely (maybe through the decay of all protons, or with ‘heat death’ due to Entropy?...). Anyhow, one should note that the so-called Steady-State model, wherein the Universe apparently had no beginning, has always existed, and shall continue to exist forever, is, at present, rejected. (This observation is crucial to my argument below, as you shall see.)

Therefore, given this hypothesis that our Universe has a finite life-span, let us now choose a value for it (which may be increased as much as one wants without affecting the validity of the logic behind my example): say, 100 billion years.

Now 100 billion years $\approx (1 \times 10^{11}) \times 365.25 \times 24 \times 60 \times 60$ seconds $= 3.15576 \times 10^{18}$ seconds.

Consider the 89th Mersenne prime $M_{89} = 2^{89} - 1 \approx 6.189700196 \times 10^{26}$, and define:

[illegible]

Now $f_1\text{Hz}$ and $f_2\text{Hz}$ obviously can occur as sound-phenomena, namely ‘pitches’. Yet because M_{89} is a prime number, the *highest* frequency $f_0\text{Hz}$ (as previously defined) whose harmonic series embraces both $f_1\text{Hz}$ and $f_2\text{Hz}$ will be $f_0\text{Hz} = 0.000000000000000000000001\text{Hz} = 1 \times 10^{-24}\text{Hz}$.

Recalling that ‘frequency’ belongs to the time domain, and that ‘time’ resides (we believe) only within our Universe, I contend that $f_0\text{Hz}$ *does not exist* ... although one may describe it symbolically, via mathematics. $f_0\text{Hz}$ *can never manifest itself physically in our cosmos*, since *one cycle* of $f_0\text{Hz}$ requires 10^{24} seconds, the life-span of approximately 320,000 100-billion-year Universes! (In Hindu cosmology, which posits an endless cycle of Universes, such a vast duration would be designated as one “Day of Brahma” or thereabouts.) Now even if one were to argue that ‘time’ might not be confined to just our space-time continuum, it is quite clear that the Universe’s End would pose a very considerable interruption to the first wavelength of $f_0\text{Hz}$, thereby negating it. (For the same reason, 0Hz , a condition of perfect and everlasting stasis, cannot truly be said to exist either. 0Hz is always disrupted by *something*: subatomic quantum activity; the ubiquitous cosmic background radiation [a microwave frequency thought to be the halo-remnant of the Big Bang itself].)

All of which has some profound ramifications for music. Firstly, it seems that the Pythagoreans were right in affirming that *music is cosmic* – something that tends to be overlooked in this postmodern era. For in order to answer a relatively straightforward musical question, it has proven necessary for us to consider cosmological matters! Secondly, we have demonstrated that each finite set of frequencies Ω – and so most pieces of music – belong to some harmonic series. If a composition (irrespective of its ‘tonal organization’) is merely the articulation of elements of a *single* harmonic series, why, then, do theorists persist with so many different types of musical analysis?...